MASERS: A Python package for statistical equilibrium calculations applied to masers

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Aim

Investigation of pumping mechanisms creating inversion in level populations for observed maser transitions

Sufficiently number of levels and take into account all processes describing population exchange

Implementing the escape probability approximation method allows the investigation of such large parameter spaces within reasonable computational time

Escape probability rate equation

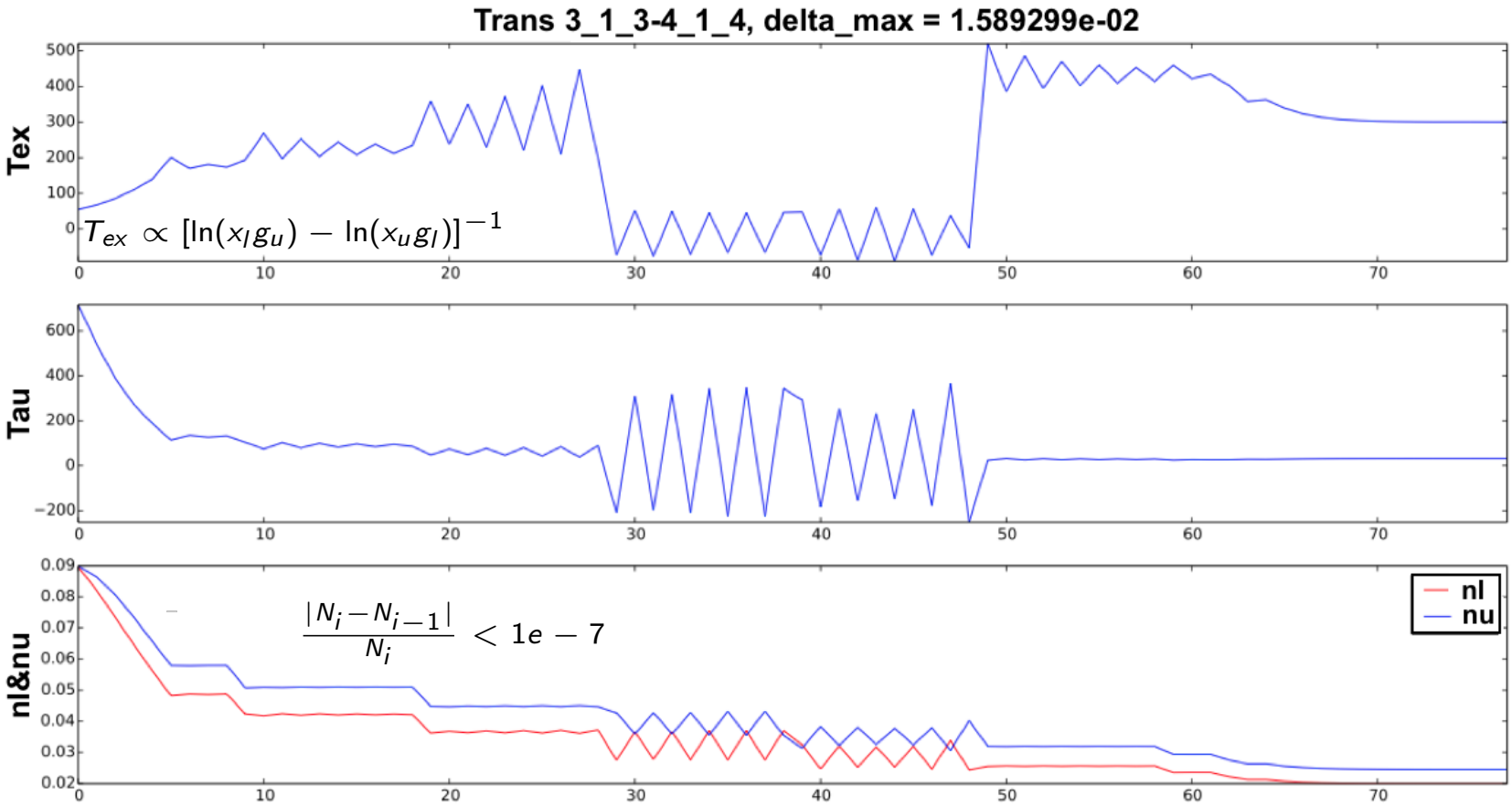
$$\begin{aligned}\frac{dn_i}{dt} &= -\sum_{j<i}\left\{A_{ij}\beta_{ij}\left[n_i + X_{ij}(n_i - n_j)\right] + C_{ij}\left[n_i - n_j \exp\left(\frac{-h\nu_{ij}}{kT}\right)\right]\right\} \\ &+ \sum_{j>i}\frac{g_j}{g_i}\left\{A_{ji}\beta_{ji}\left[n_j + X_{ji}(n_j - n_i)\right] + C_{ji}\left[n_j - n_i \exp\left(\frac{-h\nu_{ji}}{kT}\right)\right]\right\}\end{aligned}$$

$$R_{ij} = \begin{cases} A_{ji}\beta_{ji}(1 + X_{ji}) & i < j \\ A_{ij}\beta_{ij}\left(\frac{g_i}{g_j}\right)X_{ij} & j > i \\ -\sum_{i\neq j} R_{ji} & j = i \end{cases} \qquad \widetilde{C}_{ij} = \begin{cases} C_{ji} & j > i \\ -\sum_{i\neq j} C_{ij} & j = i \end{cases}$$

$$X = I_{bb} + [I(T_{\ell d}) + w_d I(T_{xd})](1 - e^{-\tau_d}) + w_{HII} I(T_e)(1 - e^{-\tau_{HII}})$$

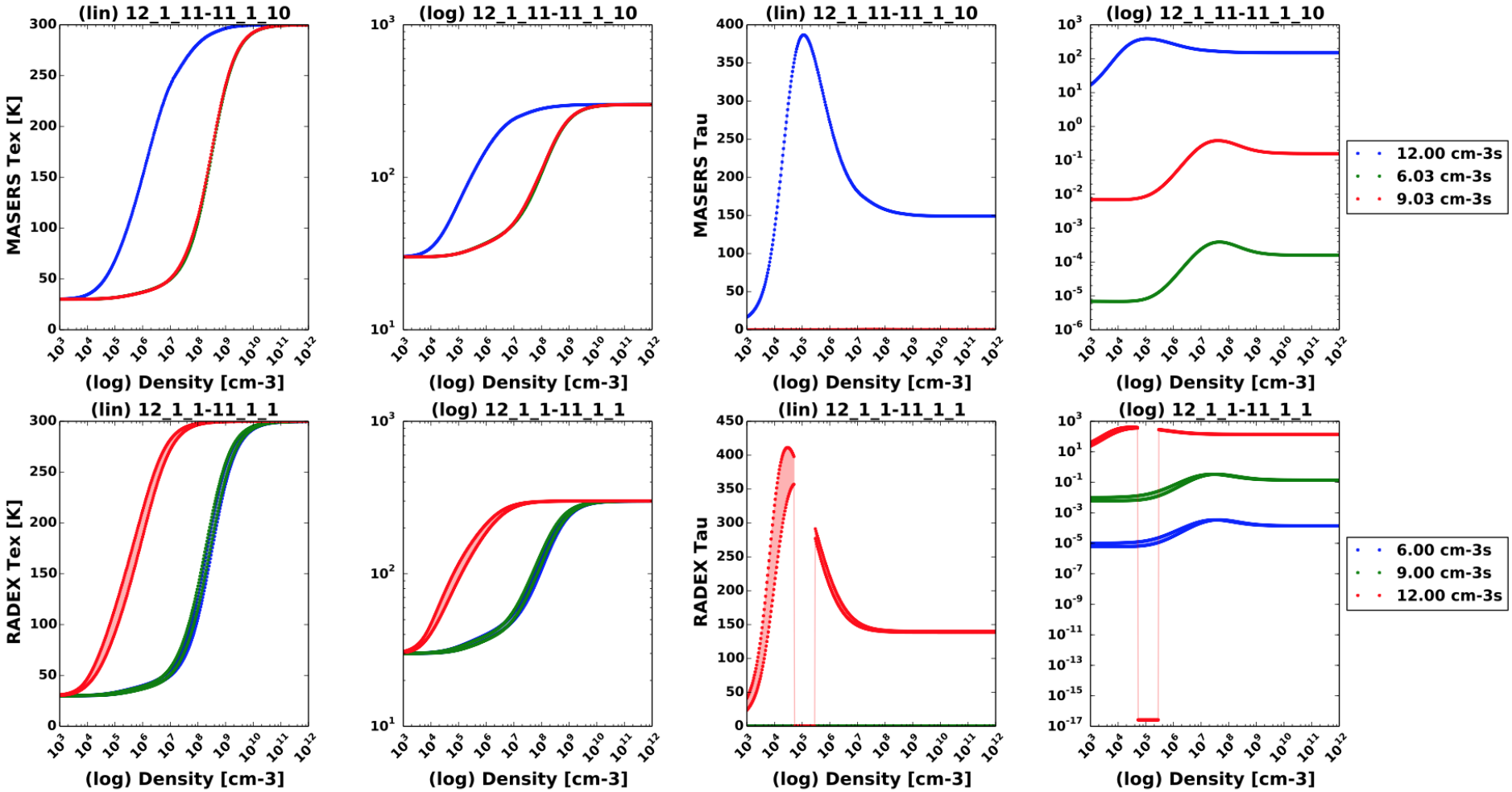
$$(\mathbf{R} + \widetilde{\mathbf{C}})\mathbf{n} = \mathbf{b}$$

Numerical limitation and instabilities

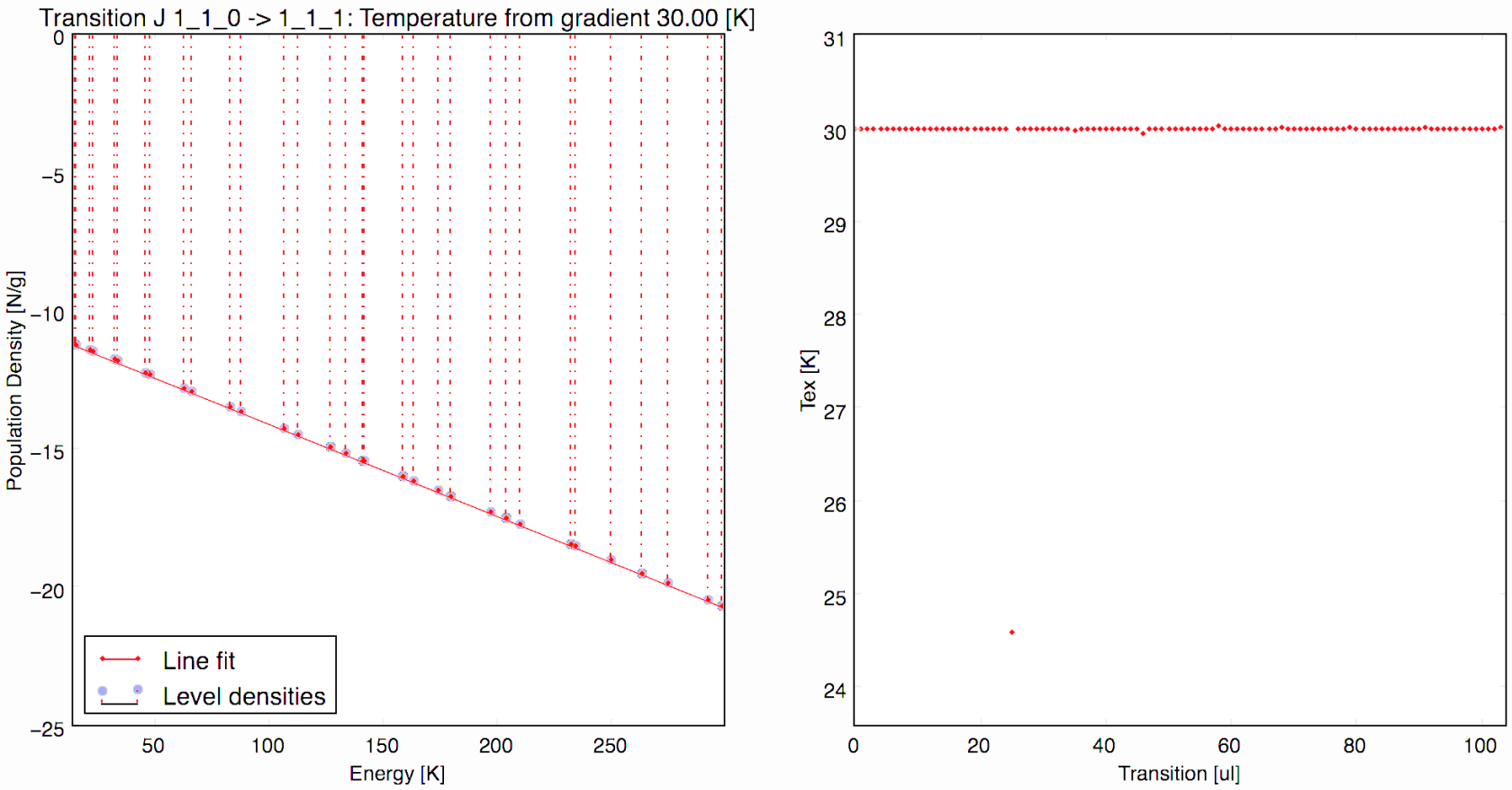


Oscillating behaviour seen in convergence evaluation using excitation temperature evaluation. The convergence showing 80 iterations over total density range $n_{H_2} = 10^6 - 10^8\,\mathrm{cm}^{-3}$ and specific column density $\frac{N_{col}}{\Delta v} = 10^{12}\,\mathrm{cm}^{-3}\mathrm{s}$.

Numerical limitation and instabilities

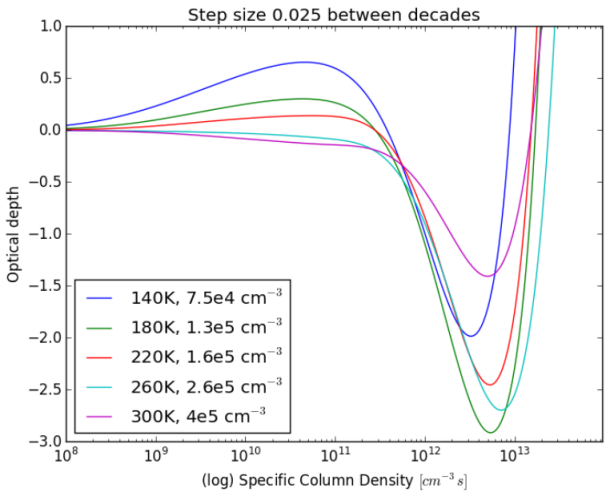


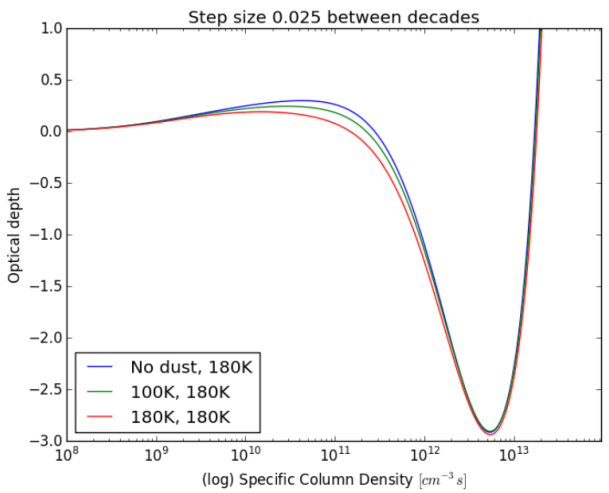
Numerical limitation and instabilities

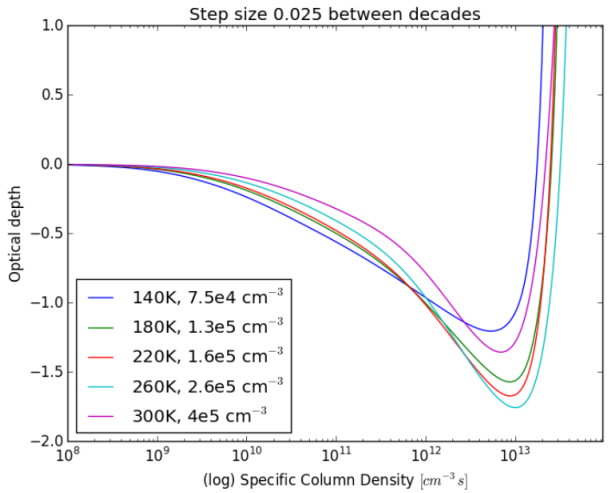


Transitions for small values of Einstein A-coefficients are forbidden

Pumping: $G37.55 + 0.20$ 4.8 *GHz* H_2CO Maser





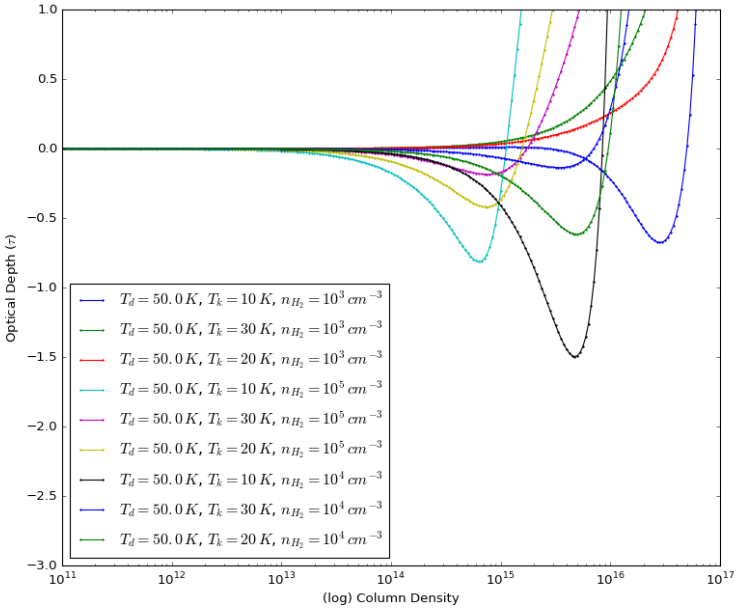


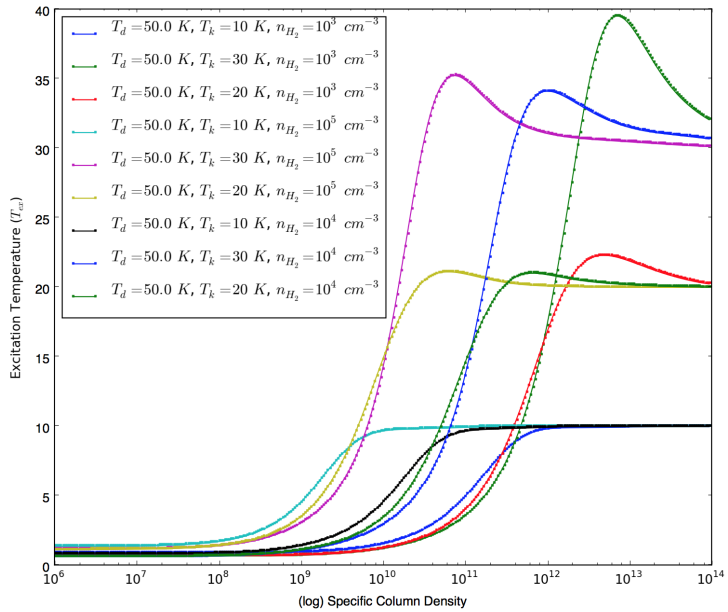
Collisional excitation showing specific column density over range 10^8–$10^{14}\,\mathrm{cm}^{-3}\,\mathrm{s}$ at a range of gas temperatures and densities

Radiative excitation via the dust continuum emission for $T_k = 180\,\mathrm{K}$, $n_{H_2} = 1.3 \times 10^5\,\mathrm{cm}^{-3}$, $w_d = 1.0$ and T_d as shown

Radiative excitation through the free-free continuum emission from an ultra- or hyper-compact *HII* region using emission measure of the *HII* region $EM = 10^{10}\,\mathrm{pc\,cm}^{-6}$, $T_e = 10^4\,\mathrm{K}$, $w_{HII} = 1.0$

False positives





Total density $n_{H2} = 10^3, 10^4, 10^5 cm^{-3}$; molecular column density $N_{h2co} = 10^{13} - 10^{18} cm^{-2}$; kinetic temperature $T_{kin} = 10, 20, 30K$
Left: using dust as a background radiation assuming $T_{bb} = 50K$,
Right: applying dust as per background radiation model $T_d = 50K$

Line Overlap

$$
\begin{aligned}
\frac{dN_i^a}{dt} &= -N_i^a \left[\sum_{j<i} A_{ij}^a \left[1-(1-f_a)(1-\beta_a)\right](1+X_{ij}^a) + \sum_{j>i} A_{ji}^a \left[1-(1-f_a)(1-\beta_a)\right](\tfrac{g_j}{g_i})X_{ji}^a \right] \\
&+ \left[\sum_{j<i} N_j^a A_{ij}^a \left[1-(1-f_a)(1-\beta_a)\right](\tfrac{g_i}{g_j})X_{ij}^a + \sum_{j>i} N_j^a A_{ji}^a \left[1-(1-f_a)(1-\beta_a)\right](1+X_{ji}^a) \right] \\
&+ N_i^a \left[\sum_{j<i} A_{ij}^a f_a(1-\beta_T)(x_a+X_{ij}^a) + \sum_{j>i} A_{ji}^a f_a(1-\beta_T)(\tfrac{g_j}{g_i})X_{ji}^a \right] \\
&- \left[\sum_{j<i} N_j^a A_{ij}^a f_a(1-\beta_T)(\tfrac{g_i}{g_j})X_{ij}^a + \sum_{j>i} N_j^a A_{ji}^a f_a(1-\beta_T)(x_a+X_{ji}^a) \right] \\
&+ \sum_{j<i}(1-\beta_T)x_a \left(\sum_{\alpha}^{\alpha\neq a} f_\alpha A_{ul}^\alpha N_u^\alpha \right)_{ij} - \sum_{j>i}(1-\beta_T)x_a \left(\sum_{\alpha}^{\alpha\neq a} f_\alpha A_{ul}^\alpha N_u^\alpha \right)_{ji}
\end{aligned}
$$

$$\mathbf{R} = \mathbf{R_a} - \mathbf{R_T} + \mathbf{O}\,,$$

$\mathbf{R_a}$ non-overlap radiation matrix of line a, $\mathbf{R_T}$ overlap region of line a and $\mathbf{O}$ contribution of overlapping lines to line a

$$
R_{ij}^a = \begin{cases} A_{ji}\hat{\beta}_a(1+X_{ji}) & i<j \\ A_{ij}\hat{\beta}_a\left(\frac{g_i}{g_j}\right)X_{ij} & i>j \\ -\sum_{i\neq j} R_{ji} & i=j \end{cases}
\qquad
R_{ij}^T = \begin{cases} A_{ji}\hat{\beta_T}(x_a+X_{ji}) & i<j \\ A_{ij}\hat{\beta_T}\left(\frac{g_i}{g_j}\right)X_{ij} & i>j \\ -\sum_{i\neq j} R_{ji} & i=j \end{cases}
$$

$$
O_{ij} = \begin{cases} -(1-\beta_T)x_a\alpha_{\mathbf{ji}} & i<j \\ (1-\beta_T)x_a\alpha_{\mathbf{ij}} & i>j \\ 0 & i=j \end{cases}
\qquad
\alpha[a_u] = \sum_{\substack{\alpha_{ul} \\ \alpha\neq a}} f_{\alpha_{ul}} A_{\alpha_{ul}}
$$

$\hat{\beta}_a = [1-(1-f_a)(1-\beta_a)]$, $\hat{\beta_T} = f_a(1-\beta_T)$

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Initial line overlap verification

